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Theoretical study of the feasibility for discriminating axial and transverse stress/strain components with Bragg sensors.

Luyckx G[a].*, De Waele W[a]., Degrieck J[a].
Vlekken J[b]., Chah K[b].

[a] Department of Mechanical Construction and Production,
Ghent University, St-Pietersnieuwstraat 41, 9000 Gent, Belgium
[b] FOS&S (Fibre optic sensors and sensing systems), Cipalstraat 14, 2440 Geel

Outline

1. Optical fibre sensing
   - Introduction
   - Bragg sensors
   - Bragg peak shift
   - Polarization Maintaining fibres
2. Sensitivity of a Bragg-sensor to a more-dimensional stress-field
   - Photo elastic effect
   - Random stress field
   - Stress field dependency of the n_{eff} of the fibre
3. Examples
   - Axial stress/strain
   - Transverse stress/strain
   - Stress calculation
   - Solution?
4. conclusions

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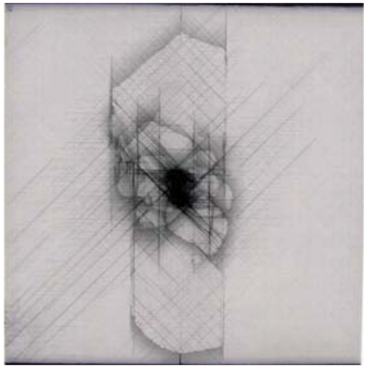
Optical fibre sensing(1/4)
Introduction

Health monitoring of composites:

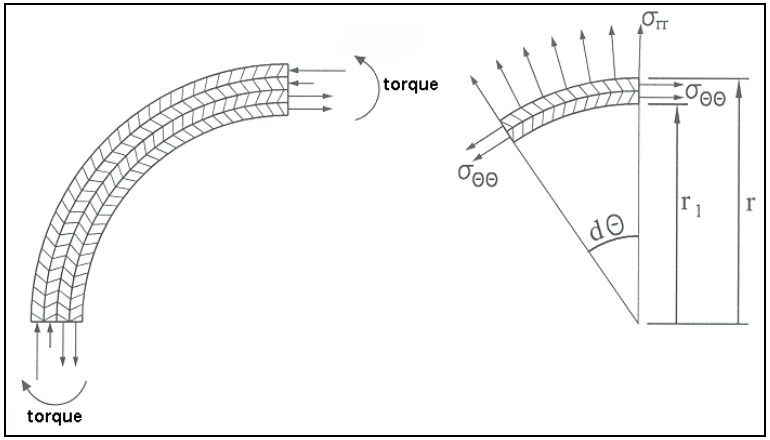


•Damage

  •Intralayer damage (matrix cracks, …)

  •Interlayer damage (delamination)

→Local damage leads to stress redistribution
→Only detectable when the entire *three-dimensional stress (or strain) state* is known.



•Through thickness stresses and strains not measurable with classical strain gauges and uni-axial FBG`s

→ Need for the development of a 3D-sensor

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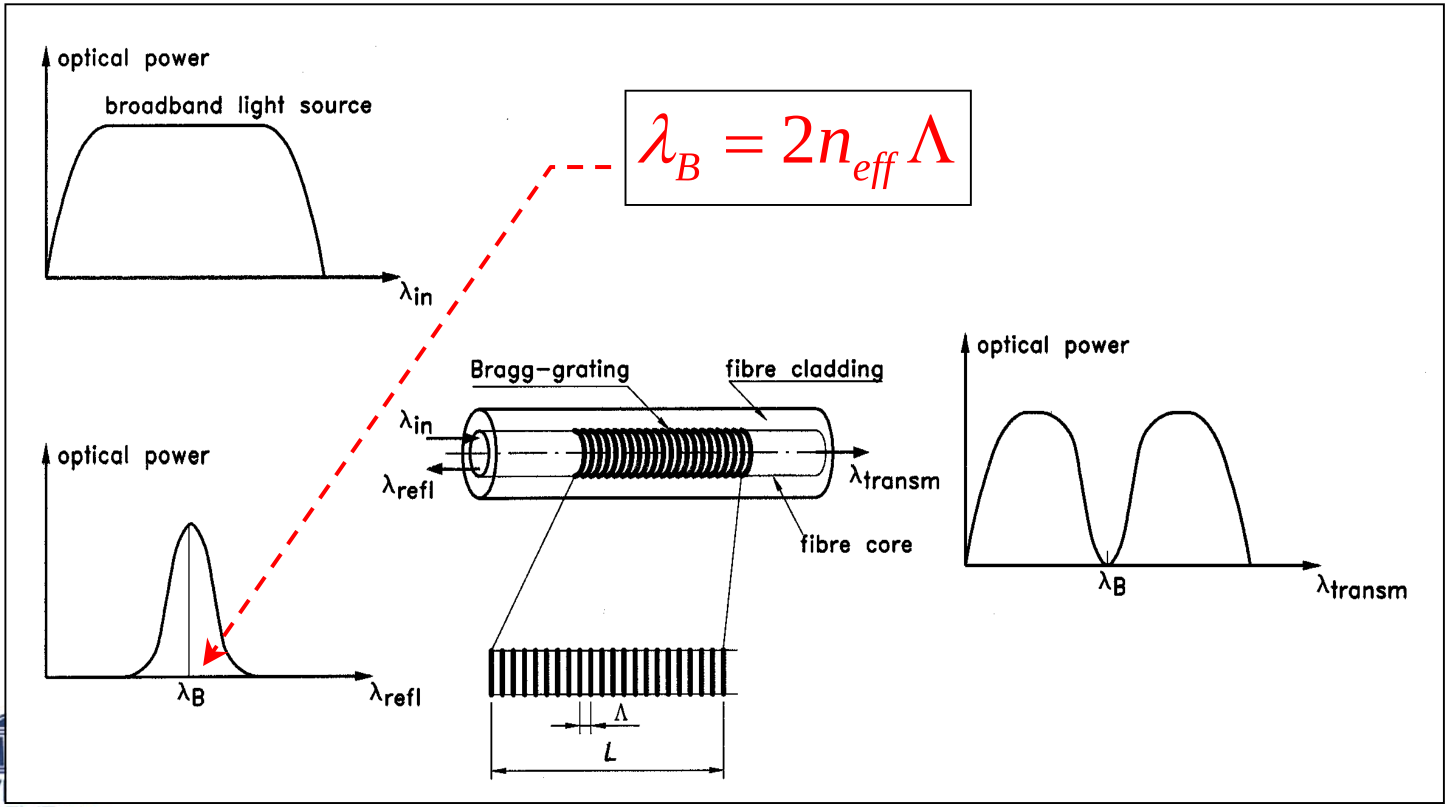
Optical fibre sensing(2/4)
Bragg-sensors

→**Optical counterpart of classical electrical strain gauges**

→**Important advantages over classical sensors:**

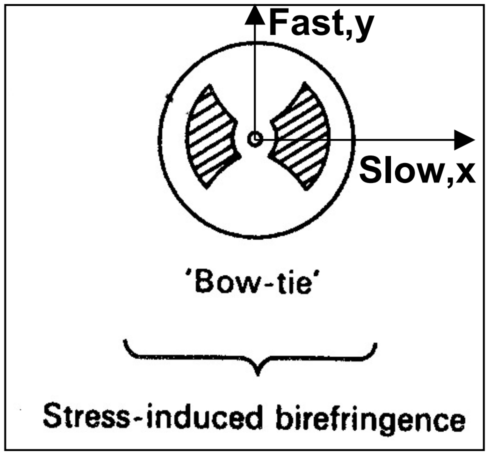
- small size (125µm)
- immune against Electro Magnetic-radiation,
- multiplexing capabilities
- temperature and fatigue resistant.



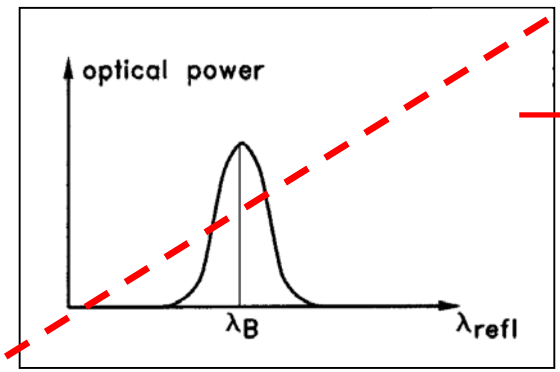
Optical fibre sensing(3/4)
Polarization Maintaining fibres

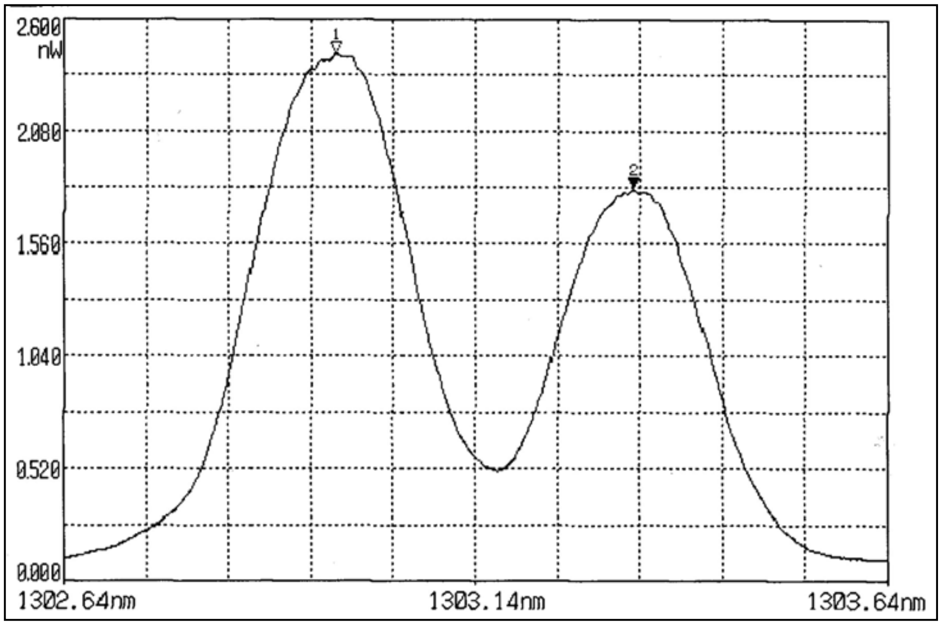
A special type of single mode optical fibre: Birefringent!

Has the ability to maintain a linear polarization state of the light.



$$\lambda_{B,slow} = 2n_{slow,eff}\Lambda$$
$$\lambda_{B,fast} = 2n_{fast,eff}\Lambda$$





Bragg-reflected spectrum for a PM-fibre

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Optical fibre sensing(4/4)

Bragg peak shift

Bragg peak shift → Pure axial loading condition($\Delta T = 0$)

$$\lambda_B = 2 n_{eff} \Lambda$$

$$\frac{\Delta \lambda_B}{\lambda_B} = (1 - P)\frac{\Delta \sigma_3}{E}$$

$$P = -\frac{1}{n_{eff}}\frac{\partial n_{eff}}{\partial \varepsilon_3}$$

Bragg peak shift → Random loading condition($\Delta T = 0$)

$$\lambda_B([\sigma]) = 2 n_{eff}([\sigma]) * \Lambda([\sigma])$$

$$[\sigma] = [\sigma_1\ \sigma_2\ \sigma_3]^T$$
$$[\sigma] = [\sigma_0] + [\Delta\sigma]$$

$$\frac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])} = \left(\frac{\partial \Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}\frac{1}{\partial \Delta\sigma_i}\Delta\sigma_i\right) + \left(\frac{\partial n_{eff}([\Delta\sigma])}{n_{eff}([\sigma_0])}\frac{1}{\partial \Delta\sigma_i}\Delta\sigma_i\right)$$

Axial strain

Change of the n_{eff} due to The photo-elastic effect

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Sensitivity to a more-dimensional stress-field(1/3)

Photo elastic effect

$$\frac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])} = \left(\frac{\partial\Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right) + \left(\frac{\partial n_{eff}([\Delta\sigma])}{n_{eff}([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right)$$

Refractive index of glass (n) coupled with the mechanical load (strain/stress) [1]

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

$p_{11} = 0.113$

$p_{12} = 0.252$ [2]

strain-optic coefficients

(Eq 1.)

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[1] A. Yariv, P. Yeh: Optical Waves in Crystals. J Wiley & Sons, 1984.
[2] Bertholds, A; Dändliker, R (1988) Journal of Lightwave Technology

Sensitivity to a more-dimensional stress-field(2/3)

Random stress field

Hooke`s Law:

$$\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix} = \frac{1}{E}\begin{Bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{Bmatrix}\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix}$$

in (Eq 1.)

$$\begin{bmatrix}\Delta\left(\frac{1}{n^2}\right)_1\\ \Delta\left(\frac{1}{n^2}\right)_2\\ \Delta\left(\frac{1}{n^2}\right)_3\\ \Delta\left(\frac{1}{n^2}\right)_4\\ \Delta\left(\frac{1}{n^2}\right)_5\\ \Delta\left(\frac{1}{n^2}\right)_6\end{bmatrix} = \frac{1}{E}\begin{bmatrix} P_1 & P_2 & P_2 & 0 & 0 & 0\\ P_2 & P_1 & P_2 & 0 & 0 & 0\\ P_2 & P_2 & P_1 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{P_1-P_2}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{P_1-P_2}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{P_1-P_2}{2}\end{bmatrix}[\sigma]$$

(Eq 2.)

stress-optic coefficients

$$P_1 = p_{11} - 2\nu p_{12}$$

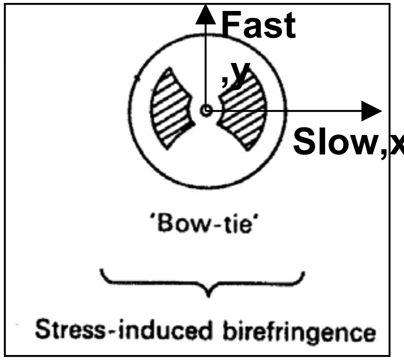
$$P_2 = -\nu p_{11} + (1-\nu)\, p_{12}$$

Sensitivity to a more-dimensional stress-field(3/3)

stress field dependency of the refractive index

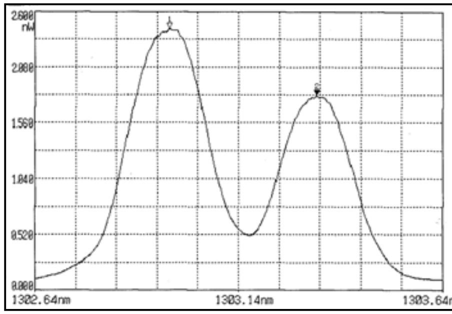
(Eq 2.)

$$\begin{cases} \Delta n_{x,slow} = -\dfrac{n_{x,slow}^{3}}{2E}\left[P_1\sigma_1 + P_2\left(\sigma_2+\sigma_3\right)\right] \\ \Delta n_{y,fast} = -\dfrac{n_{y,fast}^{3}}{2E}\left[P_1\sigma_2 + P_2\left(\sigma_1+\sigma_3\right)\right] \end{cases}$$



$$\frac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])} = \left(\frac{\partial\Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right) + \left(\frac{\partial n_{eff}([\Delta\sigma])}{n_{eff}([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right)$$

$$\begin{cases} \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{x,slow} = GF1_{\sigma,slow}\Delta\sigma_1 + GF2_{\sigma,slow}\Delta\sigma_2 + GF3_{\sigma,slow}\Delta\sigma_3 \\ \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{y,fast} = GF1_{\sigma,fast}\Delta\sigma_1 + GF2_{\sigma,fast}\Delta\sigma_2 + GF3_{\sigma,fast}\Delta\sigma_3 \end{cases}$$



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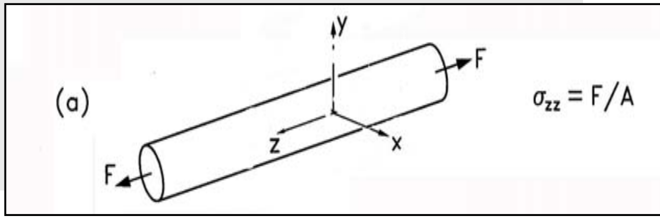
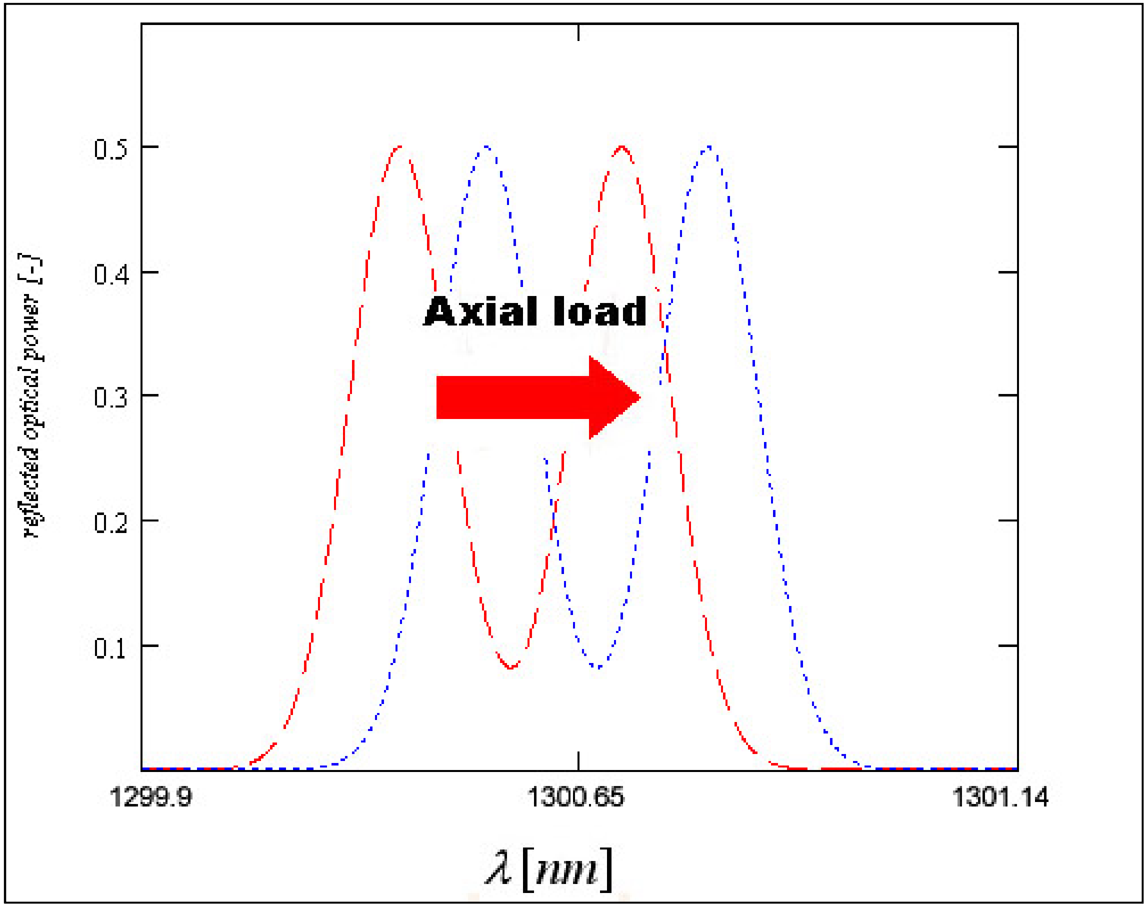
Examples(1/5)
Axial stress/strain



All stress-components are zero except for the stress-component σzz = 60N/mm².

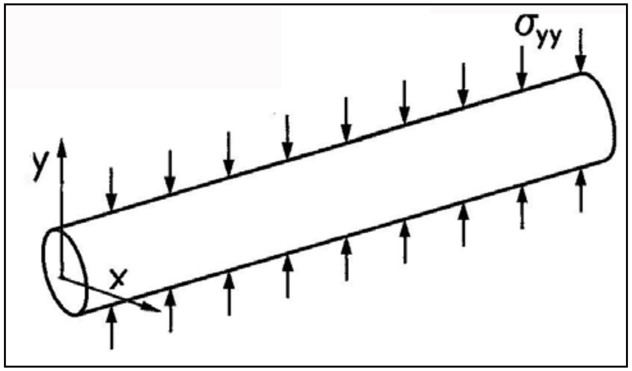


$$\left.\frac{\Delta\lambda_B}{\lambda_B}\right|_{x,slow} = \left.\frac{\Delta\lambda_B}{\lambda_B}\right|_{y,fast}$$

$$= (1-P)\varepsilon_{zz}$$

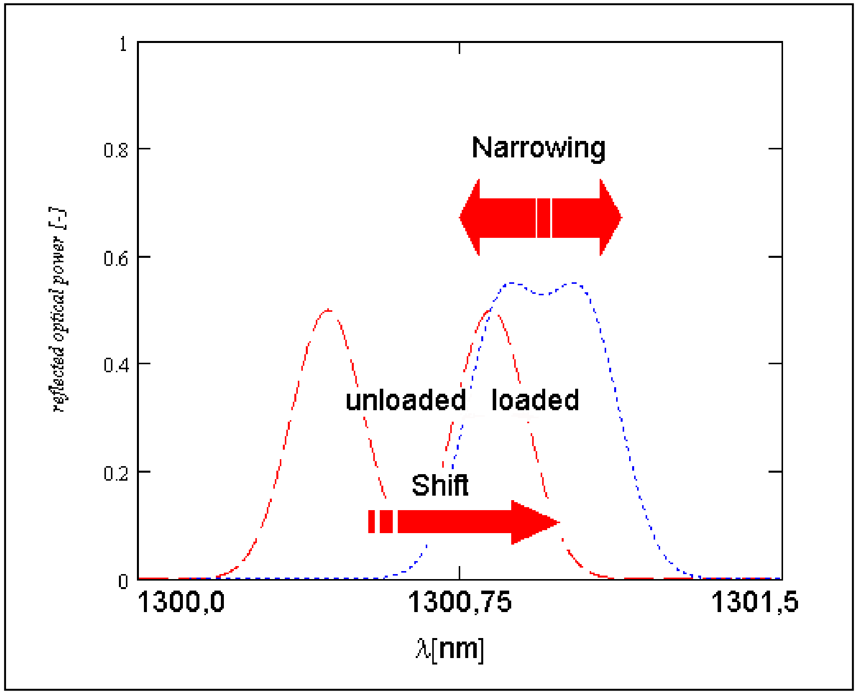
$$= 0,799\varepsilon_{zz}$$

Examples(2/5)
transversal stress/strain



The strain components: $\varepsilon_2 = \varepsilon_{yy} = \sigma_{yy}/E$

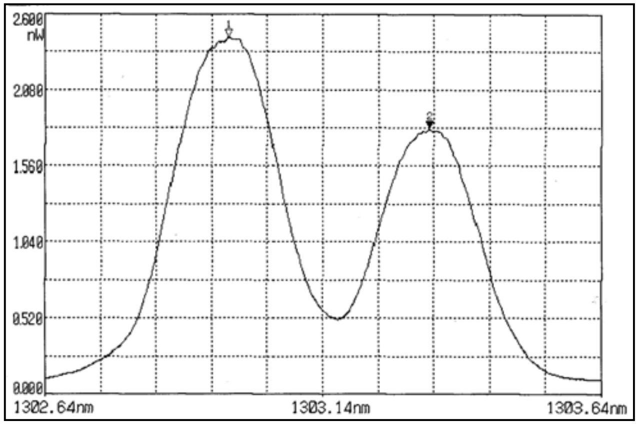
$$\begin{cases} \left.\dfrac{\Delta\lambda_B}{\lambda_B}\right|_{x,slow} = \left\{1 + \dfrac{n_{x,slow}^2}{2\nu}\left[-\nu p_{11} + (1-\nu)\, p_{12}\right]\right\}\varepsilon_{zz} \\ = 2{,}184\varepsilon_{zz} \\ \left.\dfrac{\Delta\lambda_B}{\lambda_B}\right|_{y,fast} = \left\{1 + \dfrac{n_{y,fast}^2}{2\nu}\left[p_{11} - 2\nu p_{12}\right]\right\}\varepsilon_{zz} \\ = 1{,}170\varepsilon_{zz} \end{cases}$$



Examples(3/5)

Stress calculation

$$\begin{cases} \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{x,slow} = GF1_{\sigma,slow}\Delta\sigma_1 + GF2_{\sigma,slow}\Delta\sigma_2 + GF3_{\sigma,slow}\Delta\sigma_3 \\ \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{y,fast} = GF1_{\sigma,fast}\Delta\sigma_1 + GF2_{\sigma,fast}\Delta\sigma_2 + GF3_{\sigma,fast}\Delta\sigma_3 \end{cases}$$



Three unknowns →Two equations !!!

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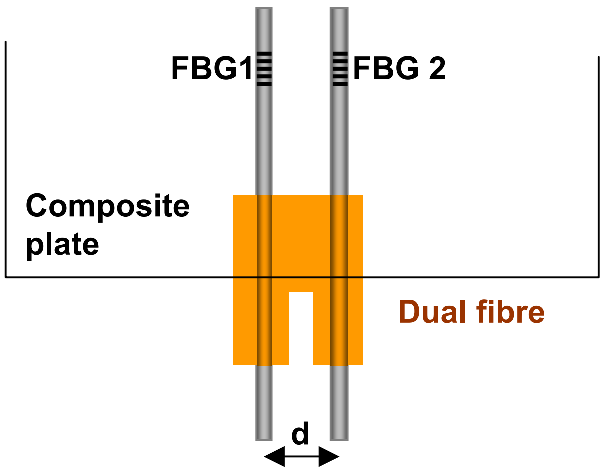
Examples(4/5)
Solution?

Two Bragg sensors THE solution?



$$\begin{cases} \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{x,slow1} = GF1_{\sigma,slow1}\Delta\sigma_1 + GF2_{\sigma,slow1}\Delta\sigma_2 + GF3_{\sigma,slow1}\Delta\sigma_3 \\ \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{y,fast1} = GF1_{\sigma,fast1}\Delta\sigma_1 + GF2_{\sigma,fast1}\Delta\sigma_2 + GF3_{\sigma,fast1}\Delta\sigma_3 \end{cases}$$

$$\begin{cases} \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{x,slow2} = GF1_{\sigma,slow2}\Delta\sigma_1 + GF2_{\sigma,slow2}\Delta\sigma_2 + GF3_{\sigma,slow2}\Delta\sigma_3 \\ \left.\dfrac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])}\right|_{y,fast2} = GF1_{\sigma,fast2}\Delta\sigma_1 + GF2_{\sigma,fast2}\Delta\sigma_2 + GF3_{\sigma,fast2}\Delta\sigma_3 \end{cases}$$

$$\begin{bmatrix} \Delta\lambda_{B1,1} \\ \Delta\lambda_{B1,2} \\ \Delta\lambda_{B2,1} \end{bmatrix} = K \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = ??$$

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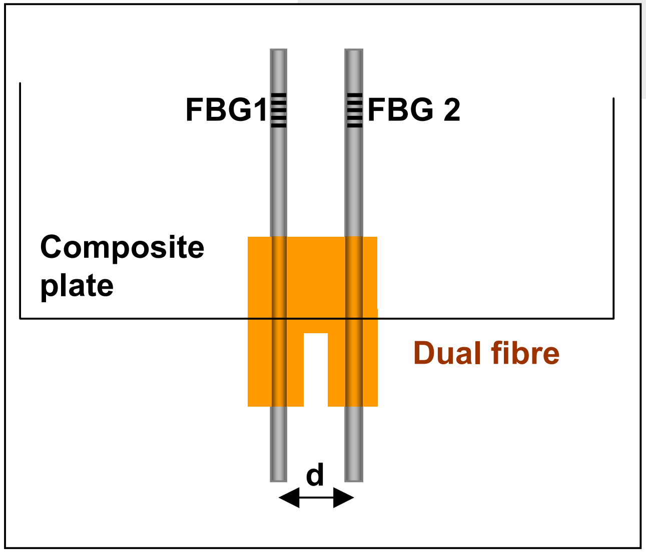
Examples(5/5)

Resolution of the Solution?



$$\begin{bmatrix} \Delta\lambda_{B1,1} \\ \Delta\lambda_{B1,2} \\ \Delta\lambda_{B2,1} \end{bmatrix} = K \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$

→Condition number of K ?
→Strain/stress resolution !!!

Inline configuration	Condition number		Error (1pm error)
Stress/Strain (3D) (Temp known)	$1,2*10^4$	Transversal 1 Strain	$3,9*10^3$ [με]
		Transversal 2 Strain	$3,9*10^3$ [με]
		Axial Strain	$2,0*10^3$ [με]

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Conclusions

The shift in Bragg-wavelength is dependent on

- **The exact load scheme (3D)**

A model was created which couples a random stress field to the Bragg peak shift of an optical fibre.

- **The effect of transverse stress causes the reflected spectrum to broaden(narrowing).**
- **The effect of axial strain is the homogenous shift of the spectrum to a higher wavelength**

Two fibres do not give a good solution for measuring the total stress/strain field(3D)!!

Need for further research!!

Acknowledgements

- The authors would like to acknowledge the financial support of the ESA (European Space Agency, project no: 19026/05/NL/IA).

- The authors also acknowledge the support of FOS&S + XenICs

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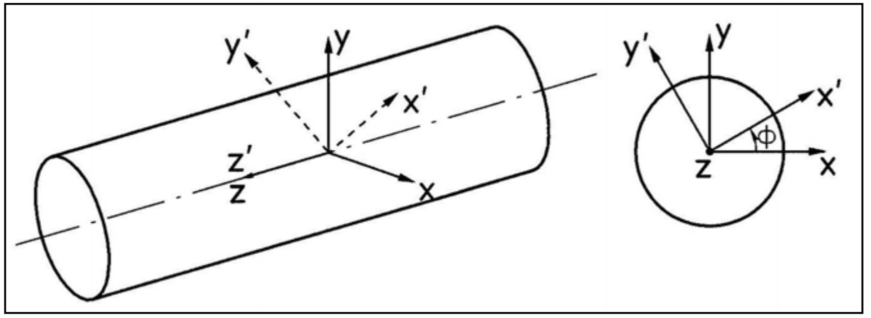
$$\Delta\lambda_B = 2\left(n_{eff}\frac{\partial\Lambda}{\partial\varepsilon} + \Lambda\frac{\partial n_{eff}}{\partial\varepsilon}\right)\Delta\varepsilon + 2\left(n_{eff}\frac{\partial\Lambda}{\partial T} + \Lambda\frac{\partial n_{eff}}{\partial T}\right)\Delta T$$

For a random orientation of the polarisation directions:

$$\begin{cases} \Delta n_{x,slow}' = -\dfrac{n_{x,slow}'^2}{2E}\left(P_1'\sigma_1 + P_2'\sigma_2 + P_2\sigma_3\right) \\ \Delta n_{y,fast}' = -\dfrac{n_{y,fast}'^2}{2E}\left(P_2'\sigma_1 + P_1'\sigma_2 + P_2\sigma_3\right) \end{cases}$$



$$\begin{cases} P_1' = P_1\cos(\phi)^2 + P_2\sin(\phi)^2 \\ P_2' = P_1\sin(\phi)^2 + P_2\cos(\phi)^2 \end{cases}$$

Sensitivity to a more-dimensional stress-field(4/6)
Bragg peak shift due to a random stress field

The Bragg-condition for an isothermal condition ($\Delta T = 0$):

$$\lambda_B([\sigma]) = 2n_{eff}([\sigma]) * \Lambda([\sigma])$$

$$[\sigma] = [\sigma_1\ \sigma_2\ \sigma_3]^T$$
$$[\sigma] = [\sigma_0] + [\Delta\sigma]$$

$$\Delta\lambda_B([\Delta\sigma]) = 2n_{eff}([\sigma_0]) * \left(\frac{\partial\Lambda([\Delta\sigma])}{\partial\Delta\sigma_i}\Delta\sigma_i\right) + 2\Lambda([\sigma_0]) * \left(\frac{\partial n_{eff}([\Delta\sigma])}{\partial\Delta\sigma_i}\Delta\sigma_i\right)$$

Substitution

$$2n_{eff}([\sigma_0]) = \frac{\lambda_B([\sigma_0])}{\Lambda([\sigma_0])}$$

$$2\Lambda([\sigma_0]) = \frac{\lambda_B([\sigma_0])}{n_{eff}([\sigma_0])}$$

$$\frac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])} = \left(\frac{\partial\Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right) + \left(\frac{\partial n_{eff}([\Delta\sigma])}{n_{eff}([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right)$$

Optical fibre sensing(4/4)

Bragg peak shift due to a random stress field

$$\frac{\Delta\lambda_B([\Delta\sigma])}{\lambda_B([\sigma_0])} = \left(\frac{\partial\Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right) + \left(\frac{\partial n_{eff}([\Delta\sigma])}{n_{eff}([\sigma_0])}\frac{1}{\partial\Delta\sigma_i}\Delta\sigma_i\right)$$

$$\partial\Delta\varepsilon_3 = \frac{\partial\Lambda([\Delta\sigma])}{\Lambda([\sigma_0])}$$

(Eq 3.)

$$\frac{\partial\Delta n`_{x,eff}([\Delta\sigma])}{n_{x,eff}([\sigma_0])} = -\frac{n`^2_{x,eff}}{2E}\partial\left(P`_1\Delta\sigma_1 + P`_2\Delta\sigma_2 + P`_2\Delta\sigma_3\right)$$

$$\frac{\partial\Delta n_{y,eff}([\Delta\sigma])}{n_{y,eff}([\sigma_0])} = -\frac{n`^2_{y,eff}}{2E}\partial\left(P`_1\Delta\sigma_1 + P`_2\Delta\sigma_2 + P`_2\Delta\sigma_3\right)$$

$$\frac{\partial\Delta\varepsilon_3}{\partial\Delta\sigma_i}\Delta\sigma_i = \frac{\partial\left(-\frac{\nu}{E}\Delta\sigma_1 - \frac{\nu}{E}\Delta\sigma_2 + \frac{1}{E}\Delta\sigma_3\right)}{\partial\Delta\sigma_i}\Delta\sigma_i$$

$$= -\frac{\nu}{E}\Delta\sigma_1 - \frac{\nu}{E}\Delta\sigma_2 + \frac{1}{E}\Delta\sigma_3$$